Section 3.3 - Rules for Differentiation

Rule 1

Derivative of a Constant Function

If f is the function with the constant value c, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

Proof of Rule 1

Find the derivative of $f(x) = x^3$

Rule 2

Power Rule for Positive Integer Powers of x

Or just the POWER RULE

If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Find the derivate of $4x^2$

a) $f(x) = x^5$

b) $g(x) = \sqrt[5]{x}$

c) $h(x) = \dfrac{1}{x^5}$

Rule 3

The Constant Multiple Rule

If u is a differentiable function of x and c, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

In ENGLISH

- If a differentiable function is multiplied by a constant, then its derivative is multiplied by the same constant

a) $y = \dfrac{3}{\sqrt[4]{x}}$

b) $y = \dfrac{3}{2x^2}$

Rule 4

The Sum Difference Rule

If u and v are differentiable functions of x, then their sum and difference are differentiable at every point where u and v are differentiable.

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

$x^2 + x$ $\quad$ x^2 $\quad$ x

Example

Find $\frac{dp}{dt}$ if $p = t^3 + 6t^2 - \frac{5}{3}t + 16$

$$p' = 3t^2 + 12t - \frac{5}{3}$$

a) $f(x) = -4x^3 - 5x^2 + 4x - 2$

b) $h(x) = -4\sqrt{x^7} - 5\sqrt[5]{x^6}$

$h(x) = -4x^{\frac{7}{2}} - 5x^{\frac{6}{5}}$

$h'(x) = -14x^{\frac{5}{2}} - 6x^{\frac{1}{5}}$

or

$= -14\sqrt{x^5} - 6\sqrt[5]{x}$

c) $y = \frac{-1}{2}\frac{-4x^2 - 4x + 3}{x^2}$

Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where?

$$y' = 4x^3 - 4x$$

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0 \quad \text{OR} \quad x^2 - 1 = 0$$

$$x = 0 \quad \text{OR} \quad x = 1 \quad \text{OR} \quad x = -1$$

Using the window [-10,10] by [-10,10], the graph of $y=0.2x^4 - 0.7x^3 - 2x^2 + 5x + 4$ has three horizontal tangents. At what points do these horizontal tangents occur?

Rule 5

The Product Rule

$f(x) = (x+2)(x-3)$

The product of two differentiable functions u and v is differentiable, and

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

OR

$$\frac{d}{dx} f(x)*g(x) = f'(x)*g(x) + g'(x)*f(x)$$

$$\frac{d}{dx}(uv) = u'v + uv'$$

Example

Find f '(x) if f(x) = (x² + 1)(x³ + 3)

$u = x^2+1 \qquad v = x^3+3$

$f'(x) = u'v + uv'$

$= (2x)(x^3+3) + (x^2+1)(3x^2)$

$= 2x^4 + 6x + 3x^4 + 3x^2$

$= 5x^4 + 3x^2 + 6x$

Rule 6

The Quotient Rule

At a point where $v \neq 0$, the quotient $y = u/v$ of two differentiable functions is differentiable, and

$$\frac{d}{dx}(u/v) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

OR

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x) * f'(x) - g'(x)*f(x)}{(g(x))^2}$$

Differentiate

$$f(x) = \frac{x^2 - 1}{x^2 + 1} \begin{matrix} = u \\ = v \end{matrix}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2} \qquad -(2x)(x^2-1)$$

$$= \frac{(2x)(x^2+1) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^3+2x - 2x^3+2x}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2} = \frac{4x}{x^4+2x^2+1}$$

Let $y = uv$ be the product of the functions u and v. Find $y'(2)$ if

$u(2) = 3$, $u'(2) = 4$, $v(2) = 1$, and $v'(2) = 2$

$$y = uv$$

$$y'(2) = u'v + uv'$$

$$= 4(1) + (3)(2)$$

$$= 4 + 6 = 10$$

Rule 7

Power Rule for Negative Integer Powers of x

If n is a negative integer and $x \neq 0$, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Examples

Find the derivatives of each

1.) $f(x) = 3x^3 - x^2 + 4x^{-1} - 2x^{-3}$

$$f'(x) = 9x^2 - 2x - 4x^{-2} + 6x^{-4}$$
$$= 9x^2 - 2x - \frac{4}{x^2} + \frac{6}{x^4}$$

2.) $f(x) = 5\sqrt{x}$ $= 5x^{\frac{1}{2}}$

$$f'(x) = \frac{5}{2}x^{-\frac{1}{2}} = \frac{5}{2x^{\frac{1}{2}}}$$
$$= \frac{5}{2\sqrt{x}}$$

3.) $f(x) = -3x\sqrt{x}$ $u = -3x \quad v = x^{\frac{1}{2}}$

$$f'(x) = u'v + uv'$$
$$= (-3)(x^{\frac{1}{2}}) + (-3x)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = -3x^{\frac{1}{2}} - \frac{3}{2}$$
$$= -\frac{9}{2}\sqrt{x}$$

4.) $f(x) = (x^3 - 2)(x + 1)$

5.) $f(x) = \dfrac{x - 3}{x^2 + 9}$

$$f(x) = \frac{\sqrt{x}}{3 + x^{-2}}$$

Finding Higher Order Derivatives

$f'(x)$

$f''(x)$

$f'''(x)$

$f^{(4)}(x)$

Find the first four derivatives of $y = x^3 - 5x^2 + 2$

Find the first five derivatives of

$$f(x) = 5x^4 - 2x^3 + x^2 + 7$$

page 124, numbers

1-6,13,14,15-22,24, 25, 29-32,33,35,37,55-58

page 126, numbers 1 - 4 all